Title: Loaded Dice and Spinners

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Most coins and dice are demonstrably (with some confidence level) fair. The asymmetry in most coins and minor manufacturing imperfections in most die do not significantly disrupt the nominally equal probabilities. In fact, significantly loaded dice are readily detected by a visually abnormal roll or spin because they are either seriously distorted or weighted. In contrast, gaming tops (used in some non-electronic role playing games) and dreidels or perinolas (six sided tops used in children's games) are very sensitive to minor imperfections even though they appear to spin normally. These spinners are spun clockwise (CW) or counter-clockwise (CCW) on their tip, slow down due to friction, then topple over and tumble to settle with one side up. Data was collected by students from commercially available toy dreidels. A hypothesis test (using the chi^2 goodness of fit test to a uniform distribution) shows that all the tested dreidels are unfair. A second hypothesis test (comparing the observed clockwise and counter-clockwise frequencies) shows that the probability distribution of all the tested dreidels is dependent on the spin direction. An ordinary differential equation (ODE) model based on rigid body dynamics and a simple static friction model is developed to describe the spin and initial fall of the top. This model explains the surprising sensitivity of these tops to the tip position and shape and the dependence on the spin direction. Finally a phase space analysis of the ODE system predicts the probability distributions (CW and CCW) for a given spinner-surface combination.