Calculus I Review

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Things to remember:

- For simplicity, the independent variable is always x in these problems. But of course it can be *any* letter. Always watch out and make sure you know what your variable is!
- Derivatives and integrals can both be "split up" when *adding*. For example,

$$\int \left(x^2 + \sin x\right)\ dx = \int x^2\ dx + \int \sin x\ dx.$$

  However, you can *not* split up when *multiplying*. For example,

$$\int x^2 \sin x\ dx \quad \neq \quad \left(\int x^2\ dx\right)\left(\int \sin x\ dx\right).$$

  The only exception to this is constants: you can always factor a number out of any derivative or integral.
- Some rules require you to break up a derivative or integral into several functions (for example, in the product rule you have to find $f(x)g(x)$). There is often more than one way to do this. The final answer will be the same no matter what choices you make, but some may be easier than others.

1. **Derivatives**
   (a) Basics – Remember these? Find these derivatives:
   - $\frac{d}{dx}(5)$
   - $\frac{d}{dx}(x^2)$
   - $\frac{d}{dx}(3x^9)$
   - $\frac{d}{dx}\left(\frac{1}{x^4}\right)$
   - $\frac{d}{dx}(\sqrt{x})$
   - $\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$
   - $\frac{d}{dx}(\sin x)$
   - $\frac{d}{dx}(\cos x)$
   - $\frac{d}{dx}(\ln x)$
   - $\frac{d}{dx}(e^x)$

(b) The product rule: $\frac{d}{dx}f(x)g(x) = f'(x)g(x) + f(x)g'(x)$.

Example: Find $\dfrac{d}{dx}(x\cos x)$.

Choose $f(x) = x$ and $g(x) = \cos x$, so

$$\begin{aligned}\frac{d}{dx}(x\cos x) &= \frac{d}{dx}[x]\cdot\cos x + x\cdot\frac{d}{dx}[\cos x] \\ &= \cos x - x\sin x\end{aligned}$$

- $\dfrac{d}{dx}(x\sqrt{x})$
- $\dfrac{d}{dx}(x^2\sin x)$
- $\dfrac{d}{dx}(4x^8\ln x)$
- $\dfrac{d}{dx}(\sin x\cos x)$
- $\dfrac{d}{dx}(e^x\sin x\ln x)$

(c) The quotient rule: $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$.

Example: Find $\dfrac{d}{dx}\left(\dfrac{\ln x}{\sin x}\right)$.

We have $f(x) = \ln x$ and $g(x) = \sin x$, so

$$\begin{aligned}\frac{d}{dx}\left(\frac{\ln x}{\sin x}\right) &= \frac{\dfrac{d}{dx}[\ln x]\cdot\sin x - \ln x\cdot\dfrac{d}{dx}[\sin x]}{(\sin x)^2} \\ &= \frac{\frac{1}{x}\cdot\sin x - \ln x\cos x}{\sin^2 x}\end{aligned}$$

- $\dfrac{d}{dx}\left(\dfrac{x}{\sqrt{x}}\right)$
- $\dfrac{d}{dx}\left(\dfrac{\sin x}{x}\right)$
- $\dfrac{d}{dx}\left(\dfrac{\sqrt{x}}{x^3}\right)$
- $\dfrac{d}{dx}\left(\dfrac{5e^x}{\ln x}\right)$
- $\dfrac{d}{dx}\left(\dfrac{1}{x^2}e^x\right)$

(d) The chain rule: $\dfrac{d}{dx}\left(f(g(x))\right) = f'(g(x))g'(x)$.

Example: Find $\dfrac{d}{dx}\left(\sqrt{x^3-2x}\right)$.

The outer function is $f(x) = \sqrt{x}$ which is more convenient to write as $f(x) = x^{1/2}$. The inner function is $g(x) = x^3 - 2x$. So

$$\begin{aligned}\frac{d}{dx}\sqrt{x^3-2x} &= \frac{d}{dx}(x^3-2x)^{1/2} \\ &= \frac{1}{2}(x^3-2x)^{-1/2}\cdot\frac{d}{dx}[x^3-2x] \\ &= \frac{1}{2}(x^3-2x)^{-1/2}(3x^2-2)\end{aligned}$$

- $\dfrac{d}{dx}\left(\sin 2x\right)$
- $\dfrac{d}{dx}\left(\sin^2 x\right)$
- $\dfrac{d}{dx}\left(\sqrt{\ln x}\right)$
- $\dfrac{d}{dx}\left(e^{3x}\right)$
- $\dfrac{d}{dx}\left(e^{\sin x}\right)$
- $\dfrac{d}{dx}\left(\sin(\cos(x))\right)$
- $\dfrac{d}{dx}\left(\ln(\sin(e^{\cos 5x^2}))\right)$

(e) Hints on taking complex derivatives: The main rule is *only use one rule at a time!* Don't try to take shortcuts by doing several rules in your head. Some complex derivatives will require many rules. In these cases, write $\dfrac{d}{dx}$ next to any derivatives you have to take as part of a rule, to remind yourself to do them later. Then go back and evaluate each derivative you have left, applying more rules as you need. For example, to find $\dfrac{d}{dx}\left(\sin(x)\cos e^x\right)$:

$$\begin{aligned}&= \cos(x)\cos(e^x) + \sin(x)\frac{d}{dx}\cos(3x^3) \\ &= \cos(x)\cos(e^x) + \sin(x)\sin(3x^3)\frac{d}{dx}3x^3 \\ &= \cos(x)\cos(e^x) + \sin(x)\sin(3x^3)9x^2\end{aligned}$$

Now go to town on these, any way you can.

Example: Find $\dfrac{d}{dx}\left(x^2e^{3x}\right)$.

$$\begin{aligned}\frac{d}{dx}x^2e^{3x} &= 2xe^{3x} + x^2\frac{d}{dx}[e^{3x}] \text{ (product rule)} \\ &= 2xe^{3x} + x^2 3e^{3x} \text{ (chain rule)} \\ &= 2xe^{3x} + 3x^2e^{3x}\end{aligned}$$

- $\dfrac{d}{dx}\left(x\sqrt{\sin x}\right)$
- $\dfrac{d}{dx}\left(e^{3x}\ln x^2\right)$
- $\dfrac{d}{dx}\left(\dfrac{\sin(3e^{\cos x})}{\ln\sqrt{x+9x^2}}\right)$
- $\dfrac{d}{dx}\left(\sin(\cos(\ln(\sin^2(x^2))))\right)$

2. **Integrals**

Here are some integrals that you should know from Calc 1.

- $\displaystyle\int 5\ dx$
- $\displaystyle\int x\ dx$
- $\displaystyle\int x^4\ dx$
- $\displaystyle\int \frac{1}{x^4}\ dx$
- $\displaystyle\int \sqrt{x}\ dx$
- $\displaystyle\int \sin x\ dx$
- $\displaystyle\int \cos x\ dx$
- $\displaystyle\int e^x\ dx$
- $\displaystyle\int \frac{1}{x}\ dx$